Algebraic Topology from a Homotopical Viewpoint

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(continued after index)
Algebraic Topology from a Homotopical Viewpoint
To my parents
To Danny

To Viola
To Sebastian and Adrian
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This book introduces the basic concepts of algebraic topology using homotopy-theoretical methods. We believe that this approach allows us to cover the material more efficiently than the more usual method using homological algebra. After an introduction to the basic concepts of homotopy theory, using homotopy groups, quasifibrations, and infinite symmetric products, we define homology groups. Furthermore, with the same tools, Eilenberg-Mac Lane spaces are constructed. These, in turn, are used to define the ordinary cohomology groups. In order to facilitate the computation, cellular homology and cohomology are defined.

In the second half of the book, vector bundles are presented and then used to define K-theory. We prove the classification theorems for vector bundles, which provide a homotopy approach to K-theory. Later on, K-theory is used to solve the Hopf invariant problem and to analyze the existence of multiplicative structures in spheres. The relationship between cohomology and vector bundles is established introducing characteristic classes and related topics. To finish the book, we unify the presentation of cohomology and K-theory by proving the Brown representation theorem and giving a short account of spectra.

In two appendices at the end of the book the proof of the Dold-Thom theorem on quasifibrations and infinite symmetric products is given in detail, and a new proof of the complex Bott periodicity theorem, using quasifibrations, is presented.

It is expected that the reader has a basic knowledge of general topology and algebra. In any case, the book is mainly aimed at advanced undergraduates and at graduate students and researchers for whose work algebraic-topological concepts are needed.

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job of Stephen Bruce Sontz, to whom we express our deep thanks. Our gratitude goes also to Springer-Verlag, particularly to Ms. Ina Lindemann for her interest in our work, and to the referees for their valuable comments which certainly helped to improve the English version of the book. Its title is, of course, a tribute to John Milnor, from whose books and papers we have learnt many important concepts, which are included in this text.

Last, but not least, we wish to acknowledge the support of Professor Albrecht Dold, who after reading the Spanish manuscript gave various important comments to make some parts better.

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